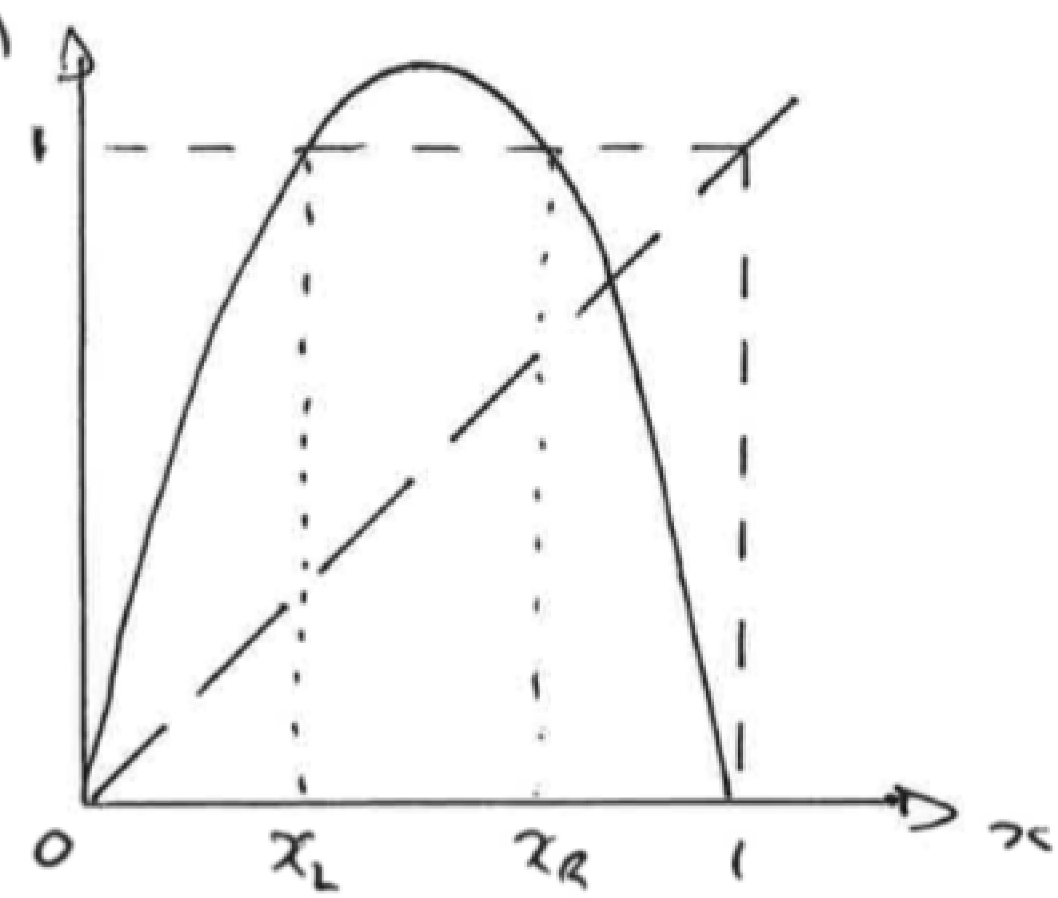
3. $I = [0,1]$. $f(x) = \mu x(1-x)$. $\mu > 2+\sqrt{5}$.

Find $f^{-1}(I)$



$F(x)=1$ if $\mu x(1-x) = 1$

$\Rightarrow \quad x - x^2 = \frac{1}{\mu}$

$\Rightarrow \quad x = \frac{1 \pm \left[1-\frac{4}{\mu}\right]^{1/2}}{2} = \underset{-}{x_L}, \underset{+}{x_R}$

Note x_L, x_R exist, and are distinct, if $\mu > 4$.

Evaluate gradient on $I \cap f^{-1}(I) = [0, x_L] \cup [x_R, 1]$

$f_x(x) = \mu(1-2x)$

On $[0, x_L]$, $f_x(x)$ takes its minimum value (which is positive) at $x = x_L$:

$$f_x(x_L) = \mu\left(1 - \left(1 - \sqrt{1-\frac{4}{\mu}}\right)\right) = \mu\sqrt{1-\frac{4}{\mu}}.$$

On $[x_R, 1]$, $f_x(x)$ takes maximum (i.e. least negative) value at $x = x_R$:

$$f_x(x_R) = -\mu\sqrt{1-\frac{4}{\mu}}.$$

Hence $|f_x(x_R)| = |f_x(x_L)| > 1$

when $\sqrt{\mu^2-4\mu} > 1$

$\Rightarrow \mu^2-4\mu-1 > 0$

Equality occurs when $\mu = \frac{4\pm\sqrt{16+4}}{2} = 2\pm\sqrt{5}.$

Since x_L, x_R require $\mu > 4$ for existence,

$|f_x(x_R)| = |f_x(x_L)| > 1$ if $\mu > 2+\sqrt{5}.$

Then, for $x \in [0, x_L] \cup [x_R, 1] = I \cap f^{-1}(I)$,

$$|f_x(x)| > |f_x(x_L)| > \lambda > 1.$$