2. Sawtooth map $x_{n+1} = 2x_n \mod 1$.

a) Let $x_0 = \frac{1}{7}$.

Then $x_1 = \frac{2}{7}$, $x_2 = \frac{4}{7}$, $x_3 = \frac{8}{7} = \frac{1}{7} = x_0$.

Hence $\frac{1}{7}$ is part of the 3-cycle $\underbrace{\{\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\}}_{\uparrow} = C_1$.

Let $x_0 = \frac{3}{7}$.

Then $x_1 = \frac{6}{7}$, $x_2 = \frac{12}{7} \to \frac{5}{7}$, $x_3 = \frac{10}{7} \to \frac{3}{7} = x_0$.

Hence $\frac{3}{7}$ is part of the 3-cycle $\underbrace{\{\frac{3}{7}, \frac{6}{7}, \frac{5}{7}\}}_{\uparrow} = C_2$.

~~By~~ The action of F corresponds to shifting the binary sequence for x one place left. So points in 3-cycles have period-3 binary expansions.

Try $\underset{\frac{1}{2}\,\frac{1}{4}\,\frac{1}{8}}{0.001}\,001\ldots = \frac{1}{8}\left(1 + \frac{1}{8} + \frac{1}{8^2} + \cdots\right)$

$= \frac{1}{8}\left(\frac{1}{1-1/8}\right) = \frac{1}{7}$.

Try $0.011\,011\ldots = \frac{1}{4} + \frac{1}{8} + \frac{1}{32} + \frac{1}{64} + \ldots$ etc

or, observe $0.011\,011\ldots = \frac{1}{7} + \frac{2}{7} = \frac{3}{7}$.

b) Let $x_0 = \frac{p}{7.2^k}$ for integers $1 \le p \le 6$, $k \geq 0$.

Then $x_1 = \frac{p}{7.2^{k-1}}$, $x_2 = \frac{p}{7.2^{k-2}}, \ldots, x_k = \frac{p}{7} < 1$

Then, if $p \in \{1, 2, 4\}$, $x_k \in \{\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\}$ and the orbit is on the 3-cycle C_1.

But if $p \in \{3, 5, 6\}$, $x_k \in \{\frac{3}{7}, \frac{5}{7}, \frac{6}{7}\}$ and the orbit is on the 3-cycle C_2.

Hence

$$O^+(x_0) = \begin{cases} \left\{ \frac{p}{7.2^k}, \frac{p}{7.2^{k-1}}, \cdots \frac{p}{7.2} \right\} \cup C_1 & \text{if } p \in \{1,2,4\} \\ \left\{ \frac{p}{7.2^k}, \frac{p}{7.2^{k-1}}, \cdots \frac{p}{7.2} \right\} \cup C_2 & \text{if } p \in \{3,5,6\} \end{cases}$$