

5.

$$f(x) = 2x, \quad G(y) = 3y \quad \text{on } [0, \infty).$$

h is a conjugacy between f and G if

$$\begin{aligned} G \circ h(x) &= h \circ f(x) \\ \Rightarrow 3h(x) &= h(2x). \end{aligned}$$

Hence one requirement is $3h(0) = h(0)$
 $\Rightarrow h(0) = 0.$

Try $h(x) = x^\alpha$ where α is to be determined.
 Then

$$\begin{aligned} 3h(x) &= h(2x) \\ \Rightarrow 3x^\alpha &= 2^\alpha x^\alpha \\ \Rightarrow 3 &= 2^\alpha \\ \Rightarrow \alpha &= \frac{\log 3}{\log 2}. \end{aligned}$$

So $h(x) = x^{\frac{\log 3}{\log 2}}$ works

Clearly $h(x)$ and $h^{-1}(y)$ are continuous on $[0, \infty)$
 So h is a homeomorphism as required.

However $h^{-1}(y) = y^{\frac{\log 2}{\log 3}}$ is not differentiable
 at $y=0$ since $\frac{\log 2}{\log 3} < 1.$

Hence h is not a diffeomorphism.