

2.

Suppose that the map F has an N -cycle $\{x_0, x_1, \dots, x_{N-1}\}$.

So $x_{n+1} = f(x_n)$ for $0 \leq n \leq N-2$
and $x_0 = f(x_{N-1})$

Let $G(x) = F^N(x)$.

$$\begin{aligned} \text{Then } G'(x) &= F'(F^{N-1}(x)) \frac{d}{dx} F^{N-1}(x) \\ &= F'(F^{N-1}(x)) F'(F^{N-2}(x)) \frac{d}{dx} F^{N-2}(x) \end{aligned}$$

Inductively

$$G'(x) = F'(F^{N-1}(x)) F'(F^{N-2}(x)) \dots F'(f(x)) F'(x)$$

If $x \in \{x_0, x_1, \dots, x_{N-1}\}$ is a point on an N -cycle then the set $\{F^{N-1}(x), F^{N-2}(x), \dots, f(x), x\}$ is just a reordering of $\{x_0, x_1, \dots, x_{N-1}\}$.

Hence $G'(x_j) = \prod_{i=0}^{N-1} F'(x_{x_i})$ for any $j \in \{0, 1, \dots, N-1\}$

So $G'(x)$ is constant on the N -cycle.