LECTURE 1 : (CHAP. 1) : DERIVING THE TRANSPORT EQ.

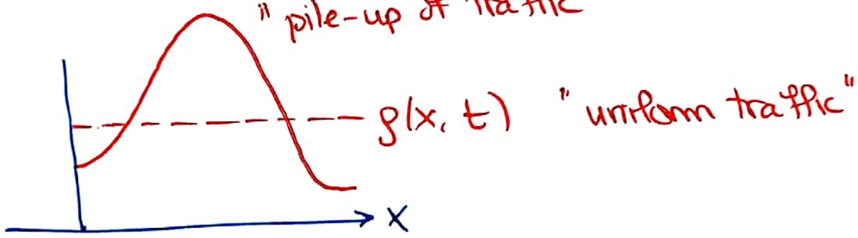
We want to motivate this course with an example.

Let $\rho(x,t)$ = density of cars at position x and time t

$[\rho]$ = # of cars per unit length.

units

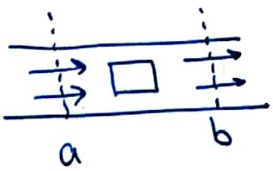
For example:



Let's consider a stretch of road $[a,b] \subset \mathbb{R}$.

$$(*) \quad \underbrace{\frac{\partial}{\partial t}\int_a^b \rho(x,t)\cdot dx}_{\text{rate of change of cars in } [a,b].} = \text{cars in at } x=a - \text{cars out at } x=b.$$

$$= q(a,t) - q(b,t)$$



where $q(x,t)$ = flow (flux) of traffic.

and $[q]$ = # of cars per unit time

$$(*) \quad \frac{\partial}{\partial t}\int_a^b \rho(x,t)\cdot dx = -\int_a^b \frac{\partial q}{\partial x}\cdot dx$$

$$\rightarrow \int_a^b \left(\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x}\right) dx = 0.$$

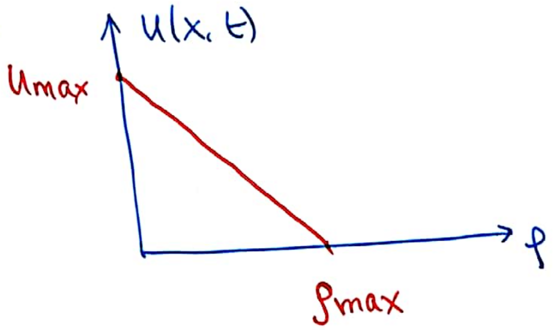
$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0}$$ since a, b is general

TRANSPORT EQUATION.

We need an empirical laws for $q(x,t)$. Note

$$q(x,t) = \underbrace{\rho(x,t)}_{\text{\# of cars per m}} \times \big(\underbrace{\text{velocity}, u(x,t)}_{m/s}\big)$$

For example



e.g. $u = u_{max} - k\rho$ $\quad k$ = constant

$$\therefore \boxed{\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0} \quad \text{where } q = \rho\{u_{max} - k\rho\}.$$

to be solved for $\rho(x,t)$.

This seems very general + powerful. We could have equally modeled.

1) Traffic, populations., biological processes.
2) heat flow, water flow
3) Electricity and magnetism
4) Stocks.

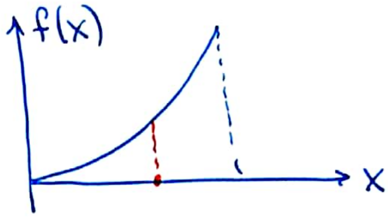
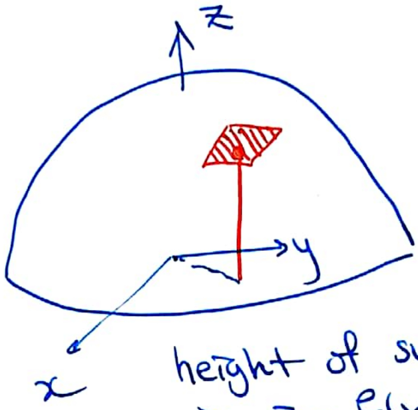
This inspires questions.

1) How do we extend to 3D (and beyond)
   Need vector calculus

2) How do we solve these equations
   (i) analytical methods
   (ii) numerical methods
   (iii) experimental methods   THIS IS THE START.

CHAPTER 2: REVIEW OF MULTIVARIABLE CALCULUS

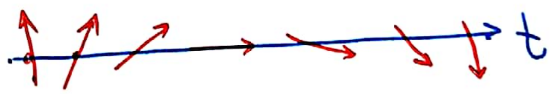
The visualisation of functions is key.

FUNCTION.	VISUAL
1D Scalar, $f: \mathbb{R} \to \mathbb{R}$. e.g. $f(x) = x^2$	
2D Scalar, $f: \mathbb{R}^2 \to \mathbb{R}$ e.g. $f(x,y) = 1 - x^2 - y^2$	 height of surface is $z = f(x,y)$
3D Scalar, $f: \mathbb{R}^3 \to \mathbb{R}$. e.g. $f(x,y,z) = x^2 + y^2 + z^2$	need 4D space but can think of every (x,y,z) having a colour, temperature.

2D Vector on a line

$\underline{v}: \mathbb{R} \to \mathbb{R}^2$

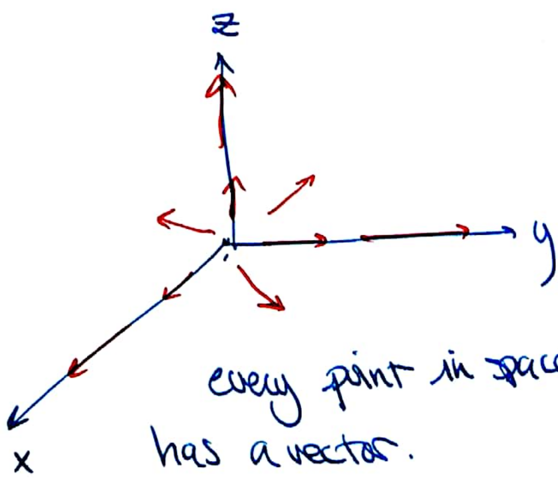
e.g. $\underline{v}(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix}$



3D Vector in $\mathbb{R}^3$

e.g. $\underline{v}: \mathbb{R}^3 \to \mathbb{R}^3$.

~~$\underline{v}(t)$~~ $=$ $\underline{v}(x,y,z) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$



every point in space has a vector.